1a. (a) True

(b) False

(c) True

(d) False

(e) True

1b. (a) True

(b) True

(c) False

(e) True

(f) True

1c. (b) {∅, {∅}} – Cardinality: 2

(c) {∅, {{∅}}, {∅}, {∅, {∅}} – Cardinality: 4

1d. (a) {-3, 0, 1, 4, 17, -12, -5, 1-, 4, 6}

(b) {1, 4}

(c) {-3, 1, 17}

(d) {-3, 0, 1, 4, 17 -12, -5, 1}

1e. (a) True

(b) False

(c) True

(e) False

(f) False

1f. (a) {aa, ab, ac, ad}

(b) {ab}

(c) {aa, ab, ac, ad}

(d) {ab}

1g. (a) Suppose that every student at a university is assigned a unique 8-digit ID number. For i ∈ {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, define the set Ai to be the set of currently enrolled students whose ID number begins with the digit i. For each digit, i, there is at least one student whose ID starts with i.

Do the sets A0, …, A9 form a partition of the set of currently enrolled students?

*We need to prove that Ai ∩ Aj is an empty set for every digit i which does not equal j.*

*We also need to prove that A ∪ Ai equals the set of enrolled students.*

We are looking for an ID number that is both in Ai and Aj, but ID numbers in Ai start with digit i and ID numbers in Aj start with digit j, and since i ≠ j, there are no ID numbers common between Ai and Aj which means that Ai ∩ Aj ≠ ∅.

Next, for an ID number of any student enrolled at the university, the ID number must start with i. We have already been told that for each digit i, there is at least one student whose ID number starts with i. Therefore, *A ∪ Ai* will be the set of enrolled students. The digit i is stated to be anywhere from one to nine (1 ≤ i ≤ 9), the sets A0, …, A9 do form a partition of the set of currently enrolled students.

1h. Domain: A, B, C, D, E.

Target: W, X, Y, Z

Range: W, Y, Z

1i. (a) X = {1, 2, 3, 4, 5} and Y = {0, 1, 2, 3, 4}. f: X → Y.

f(x) = |x – 2|

Range = {x | x ∈ Y, 0 ≤ x ≤ 3}

(b) f: {0, 1}2 → {0, 1}3. For each x ∈ {0, 1}2, f(x) = x0.

(c) f: {0, 1}2 → {0, 1}2. For each x ∈ {0, 1}2, f(x) is obtained by swapping the two bits in x. For example, f(01) = 10.

1j. (a) *f(x) = sqrt(x)*

For any negative real number, f(x) is not defined.

Therefore f(x) is not a function from **R** to **R.**

(b) *f(x) = 1/(x2-4)*

At either x = 2 or x = -2, f(x) is not defined.

Therefore f(x) is not a function from **R** to **R.**

(c) *f(x) = sqrt(x2)*

f(x) is a function whose range is [0, ∞).

1k. (a) R → R. f(x) = x2 is neither one-to-one or onto.

Example: f(8) = 64

f(-8) = 64

-8 ≠ 8

(b) R → R. g(x) = x3 is one-to-one and onto.

(c) Z → Z. h(x) = x3  is one-to-one but not onto.

Example: h(3) = 27 but cbrt(3) is not in the set of integers (Z), thus not onto

(e) Z → Z, f(x) = 5x – 4 is one-to-one but not onto.

Example: Let 5n – 4 = 0, n = 4/5, if x = 0, then f(x) = 4/5 which is not an integer

1l. (a) f: {0, 1}4→{0, 1}3 is onto but not one-to-one.

Example: f(0011) = f(1011) = 011 and 011 ≠ 1011 (not one-to-one)

1m. (e) f: P(A) → {0, 1, 2, 3, 4, 5, 6, 7, 8}. For X ⊆ A, f(X) = |X| is onto and one-to-one.

1n.(a) Z → Z f(x) = x + 3 has a well defined inverse.

y + 3 = x, y = x – 3 is an integer therefore f-1(x) = x-3 exists, making f-1(x) well defined.

1o. (e) As f(x) is both onto and one-to-one, f-1(x): P(A) → P(A) not only exists but is well defined.

2. Let A = {1}

Let B = {2}

Let q = P(A) = {{}, {1}}

Let r = P(B) = {{}, {2}}

Let s = P(A ∪ B) = {{}, {1}, {2}, {1, 2}}

|q + r| ≠ |s|

{{}, {1}, {2}, {1, 2}} ≠ {{}, {1}, {2}}

3. Let i be any number in A.

If i ∈ P(A) then i ⊆ A, therefore i ⊆ A ∪ B and i ∈ P(A ∪ B).

4a. All positive numbers in set A will end in 9 because multiplying any positive number by 10 makes that positive number equal or greater than 10. For instance, 5 \* 10 = 50, and then subtract 1 equals 49. Any negative number in set A, will have the last digit as one. This means that for all numbers in set A, those numbers will be odd. The B set, similarly, will also be a set consisting of odd numbers entirely. This was actually defined in the note below this question, so. Lastly, for set C, all positive numbers will either end in 9 or 4, and all negative numbers will end in either 1 or 6. Set C is a set containing alternating numbers between odd and even. Let D = (B ∩ C) results in set D consisting of all the odd numbers present in both B and C. All positive numbers in set D will end in 9 and all negative numbers will end in 1, since the set is entirely odd numbers. This proves A ⊆ (B ∩ C) because A ⊆ (B ∩ C) = A ⊆ D.

4b. B = {x ∈ R : x < 2}, C = {x ∈ R : x > −2} meaning B = (-∞, 2) and C = (-2, ∞). Any member of set A, where set A is defined as A = {x ∈ R : x2 < 4}, must be less than 2 and greater than -2 (as squaring a negative results in a positive). If we define a set D as being the result of B ∩ C, the negation implies that every number must be less than 2 and greater than -2. Since every member of set D is less than 2 and greater than -2, then A ⊆ (B ∩ C) is true because A = B .

5. If the composition g ◦ f is onto, then g must be onto. Let us define a variable n that exists in C. If g ◦ f is onto, according to our composition’s definition (x ∈ A, (g ◦ f)(x) = g(f(x))), there exists a number x where g ◦ f = n. Let us define another variable m, where m = f(x) ∈ B. If we take g(m), we find that this equals n. Therefore, g is onto.